

DIS97-1

BFKL-Regge expansion  
for the proton structure function  
at small  $x$

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Chicago, April 14-18

# 1. The problem of extrapolation of structure functions to a domain of small $x$

Extrapolations are not unique:

pre-HERA fits to  $F_{2p}(x, Q^2)$   
are equally good for large  $x$  data  
but seldom come close to the new  
data points at smaller  $x$ .

Limited predictive power of GLDAP<sub>q</sub>,  
 $\log Q^2$ -GLDAP-evolution  
breaks down at small  $x$  and  
is superceded by  
 $\log(1/x)$ -BFKL-evolution.

BFKL solution of the extr. problem:

take  $F_2(x, Q^2) = F_2^{(0)}(Q^2) \cdot (1/x)^{\Delta_0}$   
with  $\Delta_0 \approx 1$ . (for the leading singularity).  
The pomeron pole dominance at  $x \rightarrow 0$ .

$$\Delta_0 \equiv \Delta_P$$

At moderate  $x$   
 Subleading poweron poles  
 with smaller intercepts  
 can not be neglected.

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We present an evaluation of

- $\Delta_n$
- $\alpha'_n \quad \alpha_n(t) = 1 + \Delta_n + \alpha'_n t$
- $F_2^{(n)}(Q^2)$
- $F_2(x, Q^2) = F_2^{(0)}(Q^2) x^{-\Delta_0}$   
 $+ F_2^{(1)}(Q^2) x^{-\Delta_1}$   
 $+ F_2^{(2)}(Q^2) x^{-\Delta_2}$

$$\Delta_0 \approx 0.40; \Delta_1 \approx 0.22; \Delta_2 \approx 0.15$$

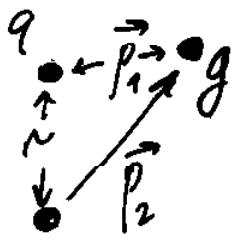
(2) BFKL eq. in color dipole representation

$$\frac{\partial \sigma_n}{\partial \xi} = K \otimes \sigma_n =$$

Nikolaev  
Zakharov  
VRZ 1994

$$= \frac{3}{8\pi^2} \int d^2 p_1 \mu_0^2 / g_s(r_i) K_1(\mu_0 p_1) \frac{\vec{p}_1}{p_1} -$$

Mueller  
Pittel '94



$$- g_s(r_2) K_1(\mu_0 p_2) \frac{\vec{p}_2}{p_2} \left[ \frac{2}{r} [\sigma_n(p_1) + \sigma_n(p_2) - \sigma_n(r)] \right]$$

$\sigma_n(r, \xi)$  - photoabsorption cross section

r - dipole size

$$\xi = \log(1/r)$$

$$\sigma_s(r) = \frac{g_s^2(r)}{4\pi}$$

Infrared cut off:  $\mu_0 = R_c^{-1}$  - inverse correlation radius of perturbative gluons

At  $\mu_0 \rightarrow 0 \Rightarrow$  the original BFKL equation

Looking for the solution with  
Regge-behaviour

$$\sigma_n(\xi, r) = \sigma_n(r) e^{\Delta_n \xi} = \sigma_n(r) (1/x)^{\Delta_n}$$

Then:

$$K \otimes \sigma_n(r) = \Delta_n \sigma_n(r)$$

1. At  $r \rightarrow 0$  and/or  $\alpha_s(r) \rightarrow 0$

$$\sigma_n(r) \sim r^2 \left( \frac{1}{\alpha_s(r)} \right)^{\gamma_n - 1}$$

Nikolaev  
Zakharov 1981

$$\gamma_n = \frac{4}{3 \Delta_n}$$

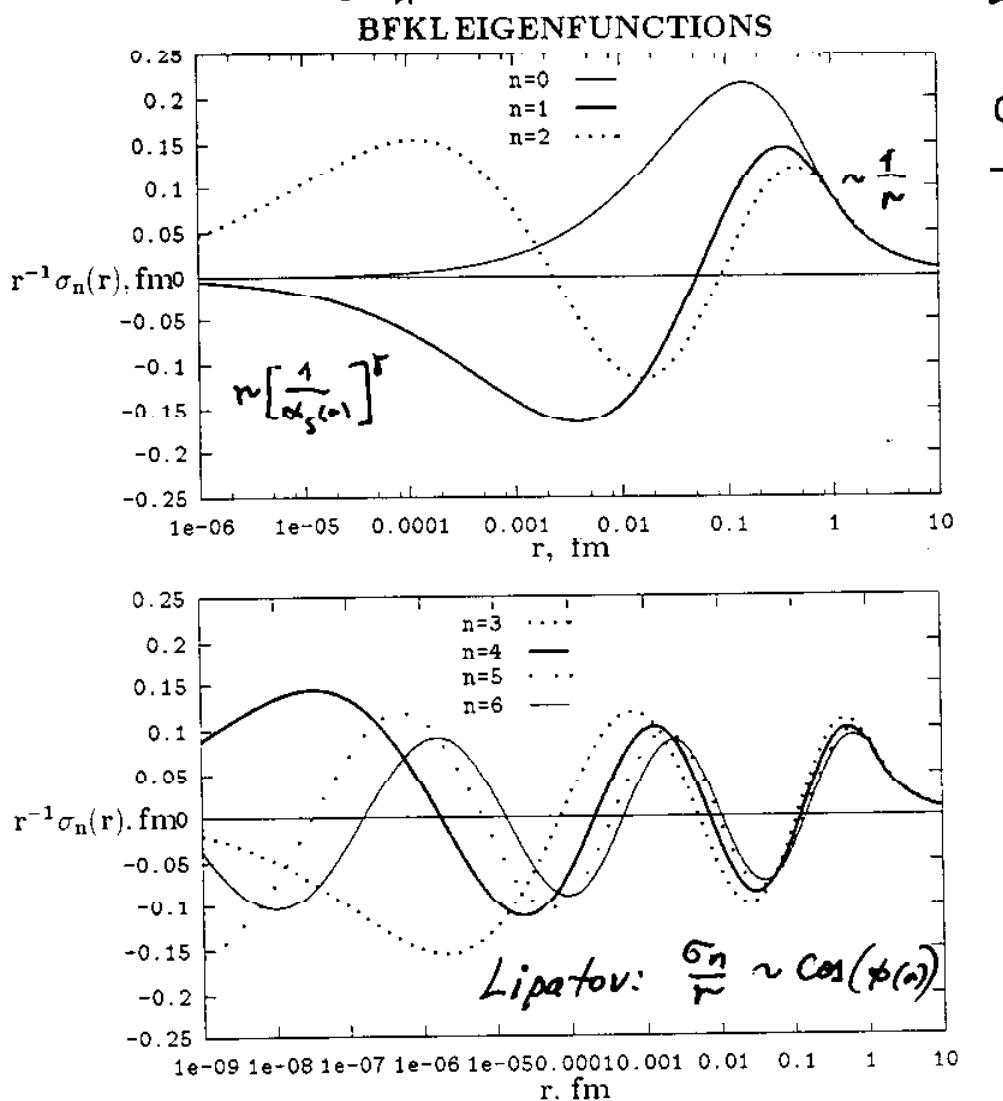
2. for  $r \gg 1/\mu_0$

$$\sigma_n(r) = \text{const}$$

From Quantum Mechanics:

- 1) Leading eigenfunction ( $n=0$ ) is node free
- 2) The  $n$ 'th solution has  $n$  nodes

A practical approach to this eigenvalue problem is a variational procedure applied to a class of the  $n$ -node polynomials  $P_n(z)$  in a variable  $z \sim [1/d_s(r)]^\delta$ .

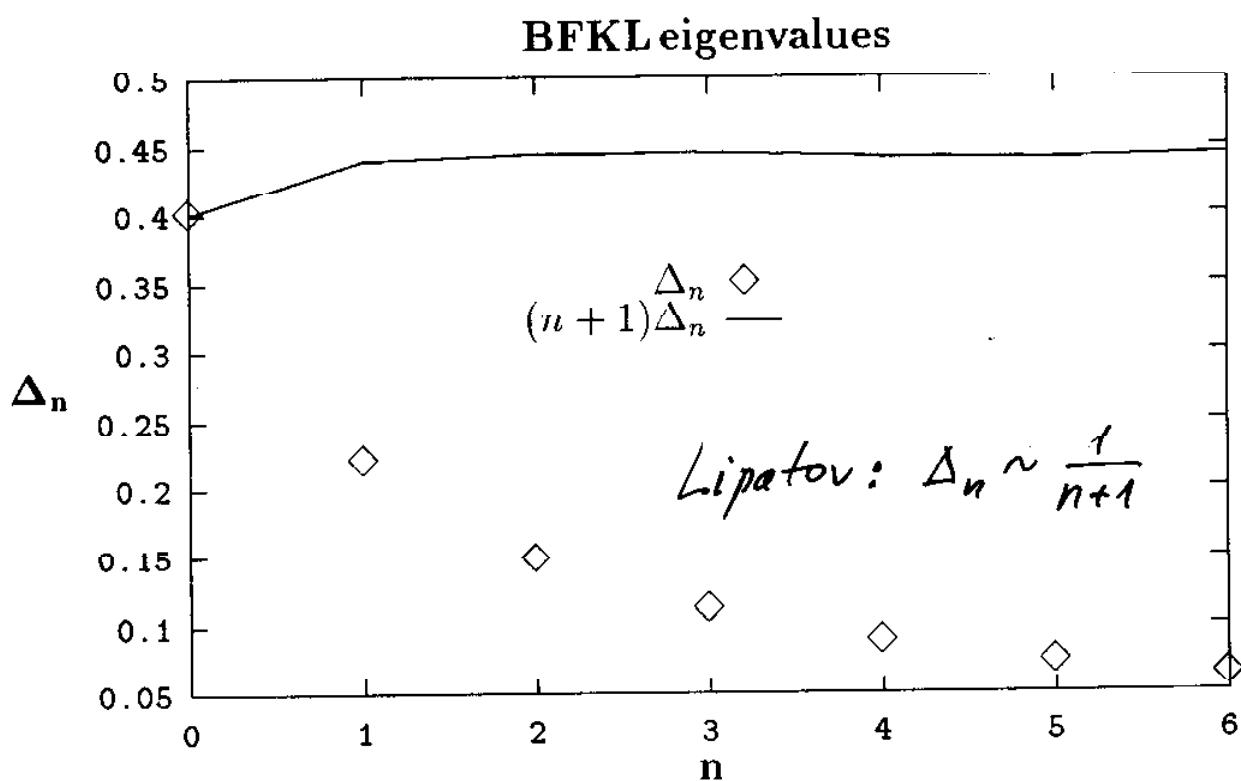


$$\sigma_n = \sigma_0(r) P_n(z)$$

Convenient quantity -  $\frac{\sigma_n(r)}{r}$ .

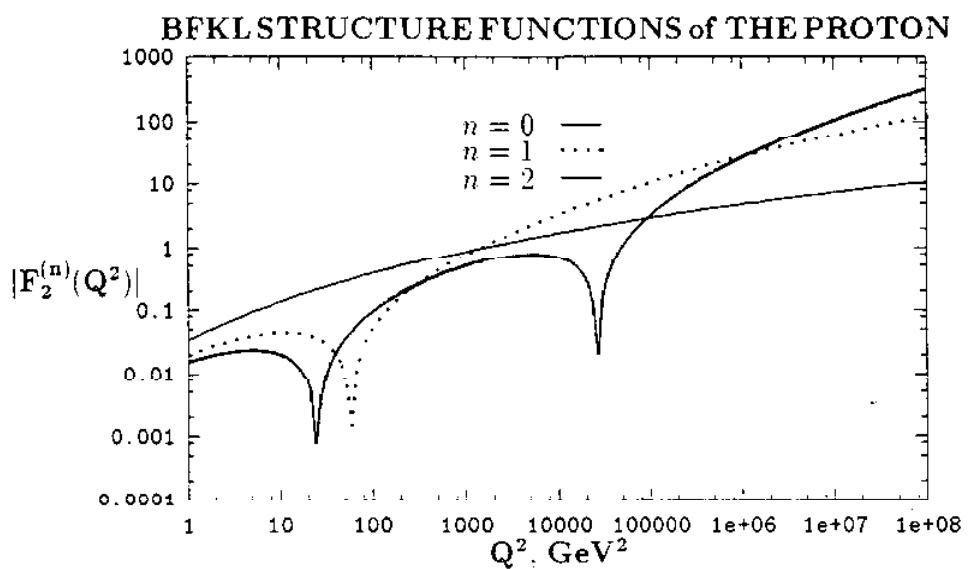
To a crude approximation  $\frac{\sigma_n(r)}{r}$  is close to Lipatov's quasiclassical eigenfunctions,  $E_n(a) \sim \cos(\phi(r))$  in nodal region

Lipatov '8



Quasiclassical consideration (Lipatov '86  
gives an estimate:

$$\Delta_n \sim \frac{1}{n+1} \quad \text{at} \quad n \gg 1$$



BFKL str. functions from dipole cross sections

$$F_2(x, Q^2) = \frac{Q^2}{4\pi \alpha_{\text{em}}} \cdot [\sigma_T(x, Q^2) + \sigma_L(x, Q^2)]$$

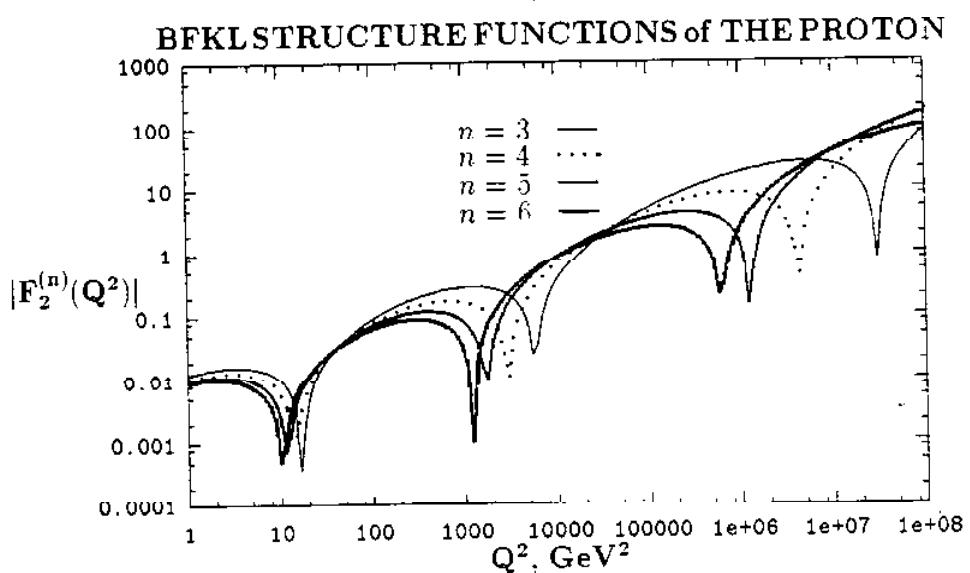
In terms of the light-cone  $q\bar{q}$  wave function  
of the photon  $\gamma_{T,L}(z, n)$

Nikolaev  
Zakharov '9-

$$\gamma_{T,L}(x, Q^2) = \int_0^1 dz \int d^2 \vec{r} / 4 \gamma_{T,L}(z, r, Q^2)^2 \sigma(x, r)$$

- At large  $Q^2$ , far beyond the nodal region,

$$F_2^{(n)} \sim \left[ \frac{1}{\alpha_s(Q^2)} \right]^{\frac{4}{3\Delta_n}}$$



- Close similarity of structure functions with  $n > 2$  at  $Q^2 \lesssim 10^3 \text{ GeV}^2$

DIS44-12

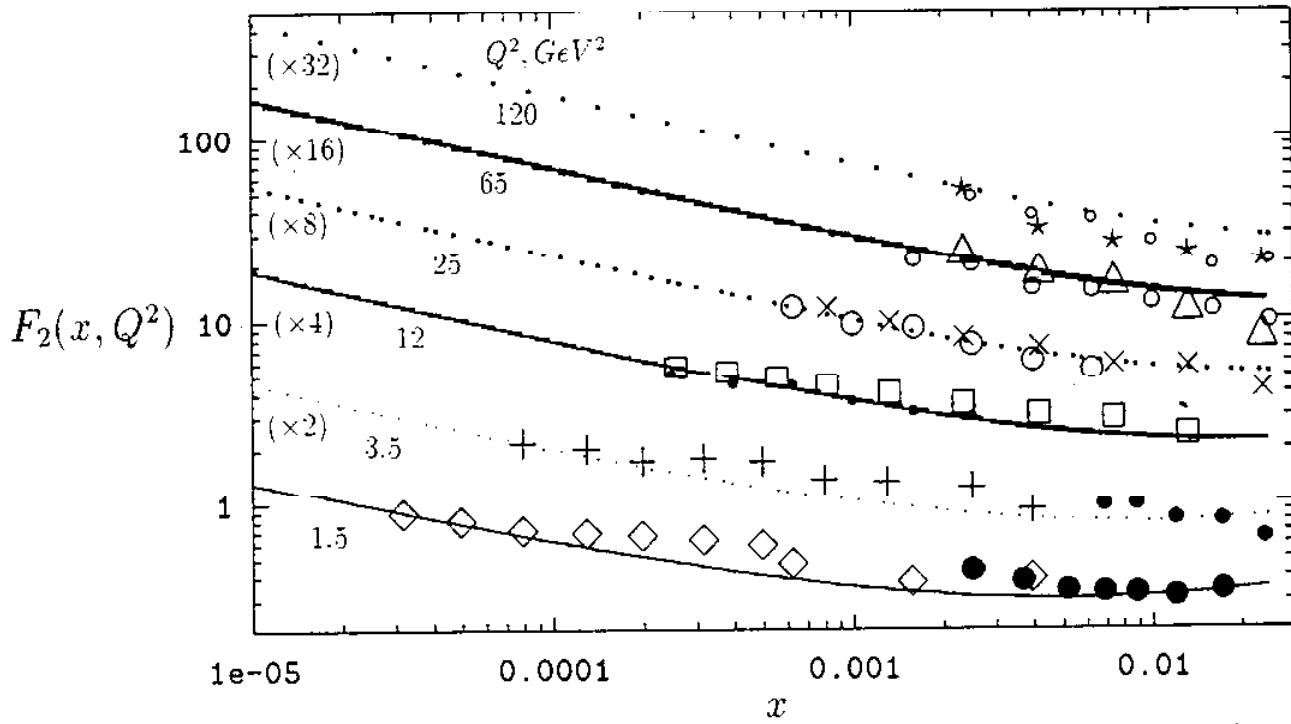
## BFKL-Regge phenomenology of DIS str. fun.

At small  $x$  only  $Q^2 \lesssim 10^3$  are accessible.

where only s.f. with  $n=0, 1, 2$  are distinct.

$F_2^{(n)}$  with  $n > 2$  are hardly distinguishable

Three pole approximation vs. H1, ZEUS and E665 data



Then the BFKL-Regge expansion for the proton structure function reads:

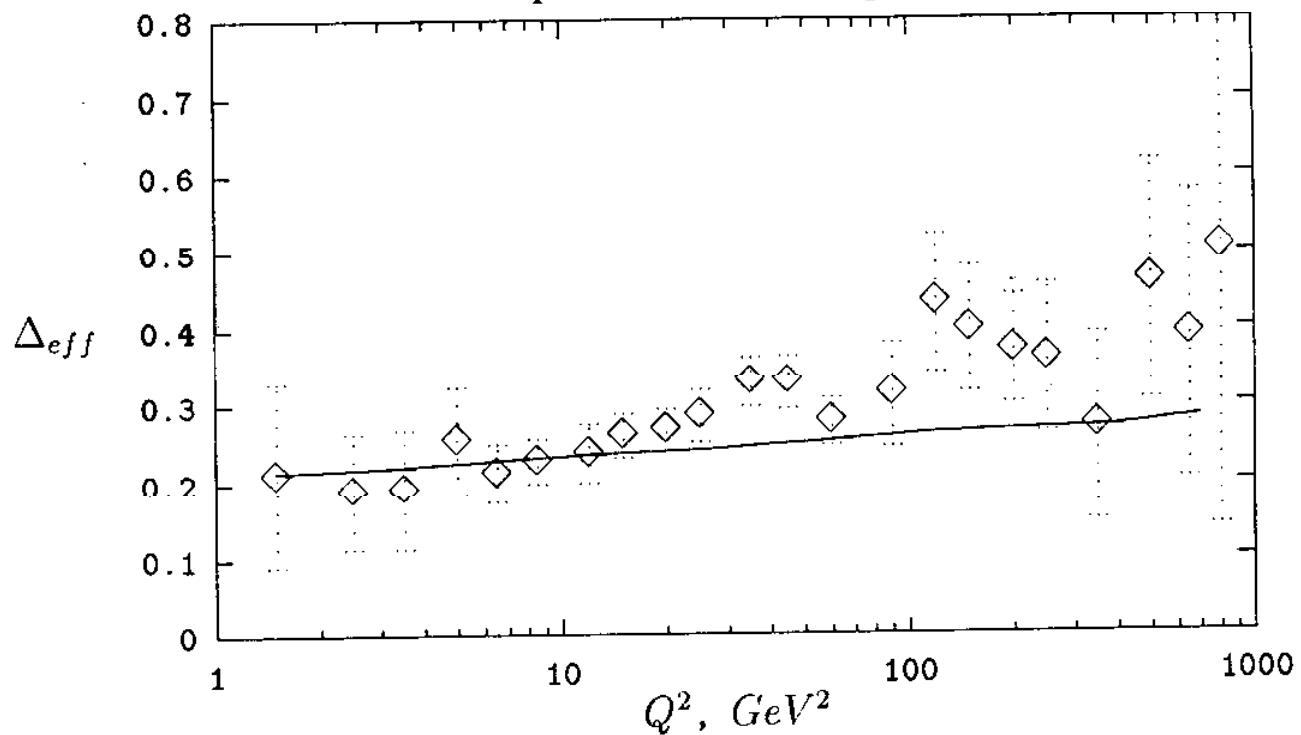
$$F_2(x, Q^2) = F_2^{(0)}(Q^2) (x_0/x)^{\Delta_0} + F_2^{(1)}(Q^2) (x_0/x)^{\Delta_1} + \\ F_2^{(2)}(Q^2) (x_0/x)^{\Delta_2} + \text{certain contr. from valence and soft}$$

$$\Delta_0 = 0.40, \Delta_1 = 0.22, \Delta_2 = 0.15$$

The Three Pole Approximation seems to exhaust the existing experimental data.

$$\Delta_{eff} = - \frac{\partial \log F_2}{\partial \ln(1/x)}$$

Effective pomeron intercept vs. H1 data



Comparison is not very instructive  
since kinematically only rather large  $x$   
are accessible at largest  $Q^2$ .

The set of pomerons:

$$\left. \begin{array}{l} \alpha_0(t) = 1 + 0.4 + 0.072 \cdot t \\ \alpha_1(t) = 1 + 0.22 + 0.066 \cdot t \\ \alpha_2(t) = 1 + 0.15 + 0.062 \cdot t \end{array} \right\}$$

### POMERON TRAJECTORY SLOPE

